Conservation

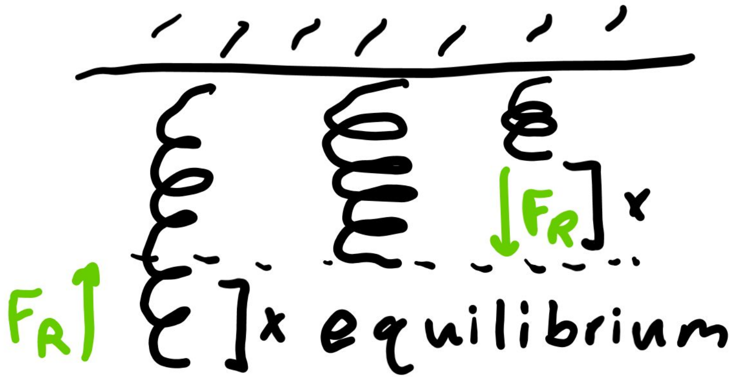
Springs

Equilibrium

Springs pull everything toward the center, called the equilibrium.

You can pull objects on a spring a distance x from the spring.

The spring will always apply a force to restore itself to equilibrium.



Hooke's law

The magnitude of the restoring force, or the force bringing it back to equilibrium is proportional to x, the distance either compressed or stretched.

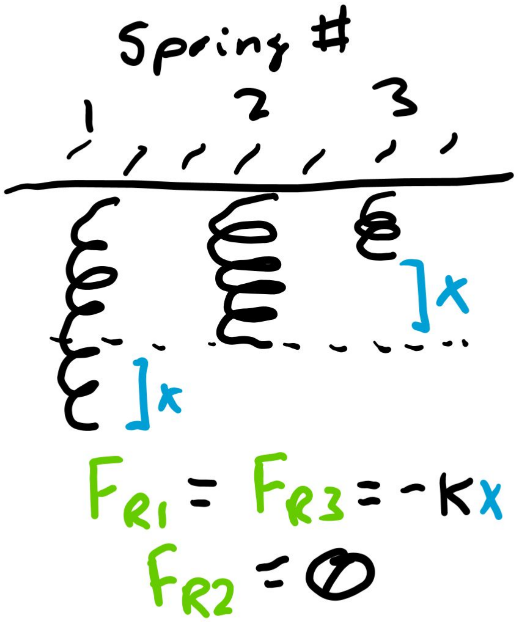
$F_r = -kx$, where k is a constant and x is the distance the spring is stretched. Negative as it's opposite the applied force which is usually denoted as positive.

Note that F_r is negative no matter if you compress or stretch it as the negative sign tells you that it's acting against your F_a used to deform it in either direction.

Spring constant

The units of the spring constant are N/m or kg/s^2 since $F_r = -kx$.

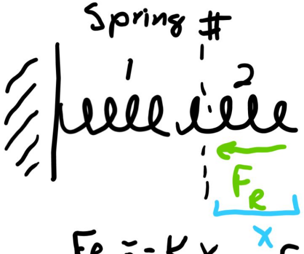
The larger the constant, the stiffer the spring, meaning it takes more force to stretch it out the same distance as one with a smaller spring constant.



Springs in series

If you stack springs, they actually seem weaker than before because the “stretching” is distributed across more springs.

The formula for the new spring constant is $k = 1/(1/k_1 + 1/k_2)$ which will always be smaller than k_1 or k_2.



Let spring 1 deform by x_1 and spring 2 by x_2. We know $x = x_1 + x_2$.

$F_R = -k_1 x_1$, $F_R = -k_2 x_2$ as F_R applies to both springs simultaneously, so

$x_1 = F_R/-k_1$, $x_2 = F_R/-k_2$, $F_R(-1/k_1 - 1/k_2) = x$

$F_R = \frac{-1}{(1/k_1 + 1/k_2)} x = -kx$, so $k = \frac{1}{(1/k_1 + 1/k_2)}$

Springs in parallel

The more springs you attach to an object, the harder it is to displace that object as more springs pull.

Thus, the spring constant of the system is larger.

The formula is $k = k_1 + k_2$ for springs put in a row.

Both springs deform x, so $F_{R1} = -k_1 x$ and $F_{R2} = -k_2 x$.
Total restoring force F_R is $F_{R1} + F_{R2} = -(k_1 + k_2)x = -kx$
so $k = k_1 + k_2$.

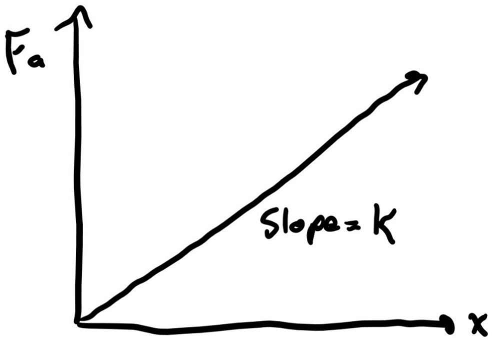
Spring potential

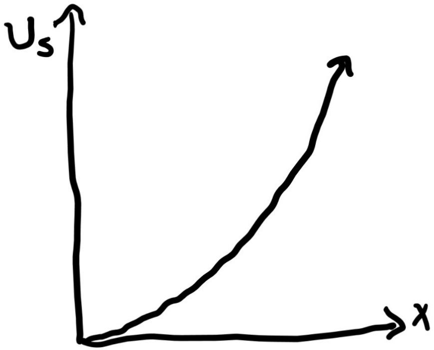
Force as you stretch the spring

As you stretch the spring over more and more distance, more and more force $F_a = -F_r$ is involved to counteract $F_r = -kx$.

Thus, the work you apply increases as you keep stretching the spring.

The energy stored in the spring as a result of your stretching is called U_s, or spring potential energy.





Integrate Hooke's law

Work is force applied over a certain distance, or in other words the integral of force with respect to distance.

The work to load a spring a distance x is thus the integral of Hooke's law $F = kx$, or $F = ½ kx^2$.

Here it's positive as $F_a=kx$ is the force we need to counteract the negative restoring force to stretch/compress the spring.

Integral approach to work

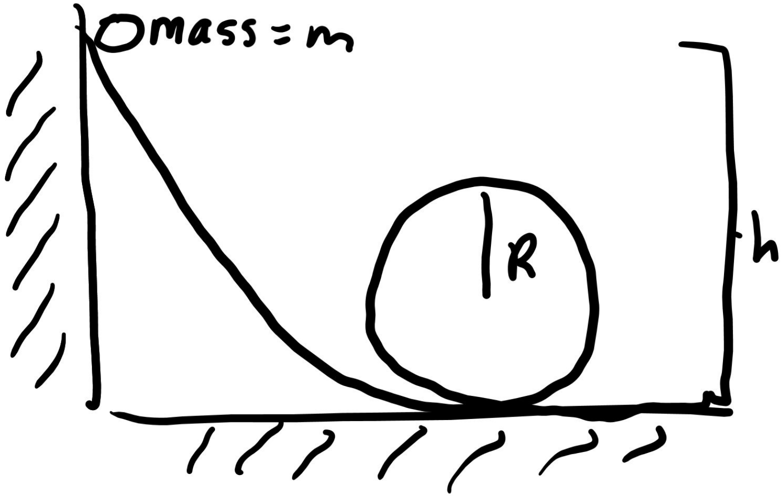
For any force function with respect to distance, like $F = 3x^2$, the work applied is the integral with respect to distance, or $W = x^3+C$.

If force is constant, then the integral $W = \int F dx = Fx$, or force times distance.

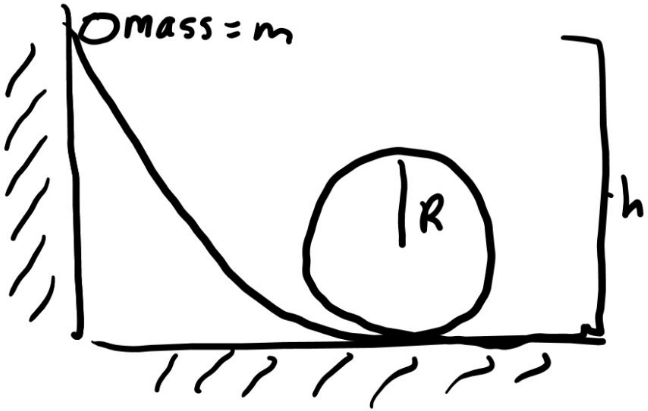
Conservation

Classic energy conservation problem

If a ball of mass m is dropped off a height of h onto a ramp that goes in a loop de loop, what must be the radius of the loop in order for the ball to be able to complete the loop without falling?



Solution



At the top, there is mgh of potential energy and at the top of the loop there is 2mgR potential energy. The rest is kinetic, so $KE = \frac{1}{2}mv^2 = mgh - 2mgR$.

$$F_c = \frac{mv^2}{R} = \frac{2KE}{R} = \frac{2mgh - 4mgR}{R}.$$

Only F_w acts as F_c at the top, so $F_c = F_w = mg = \frac{2mgh - 4mgR}{R}$.

$mg = \frac{2mgh}{R} - 4mg$, $5mg = \frac{2mgh}{R}$, so $R = \frac{2h}{5}$.

Momentum

Moving inertia

Objects in motion wish to stay in motion.

The more motion they have, and the more mass they have, the more they wish to stay moving.

Momentum is this “moving” inertia, and is defined as m*v, or mass times velocity.

Rest

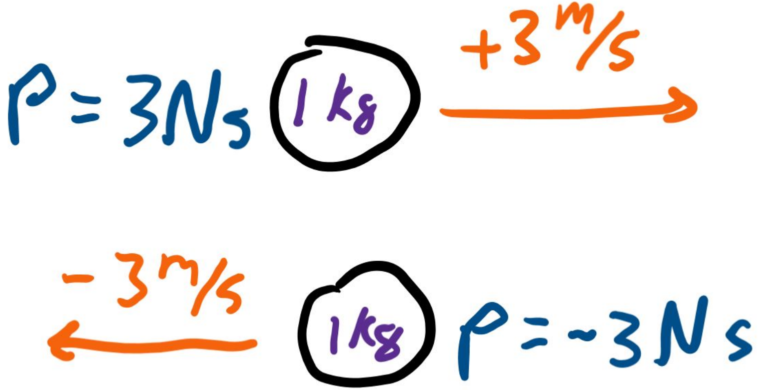
$mv = m \cdot 0 = 0$ momentum

$\longrightarrow$ motion

$p = mv$

Vector

Momentum is a vector, meaning the direction of velocity is important as you have different momentum given a different direction of motion.



Momentum is force times time

We can rewrite mv as m∫a(dt)
since a = dv/dt.

If a is constant, then this is mat = Ft,
or force times time.

Thus momentum, represented as p, is
also Ft when acceleration is constant.

If force is not constant, momentum is the
integral of force with respect to time.

$$\frac{dv}{dt} = a, \text{ so } V = \int a\,dt$$

$$\int p\,dt = \int mv\,dt = m\int v\,dt = ma = F$$

$$\text{Thus, } F = \frac{dp}{dt}$$

Integral definition

We can also integrate a force function with respect to time, like say $F = 2t$ to get $p = t^2+C$.

This is how we account for a non-constant force or acceleration.

Impulse

Impulse is change in momentum and is represented by the letter J.

If we go from 10 Ns of momentum to 5 Ns of momentum in 5 seconds, impulse is -5 Ns.

Force is change in momentum over time, or impulse divided by time, so force is -1 N.

$p = mv = 10\,kg\,m/s = 10\,Ns$

$p' = 5\,Ns$

$\Delta p = p' - p = -5\,Ns = J$

$\frac{\Delta p}{\Delta t} = \frac{J}{\Delta t} = F$

Newton’s second law

The most general form of Newton’s second law actually says $F = dp/dt$, or force is change in momentum over time.

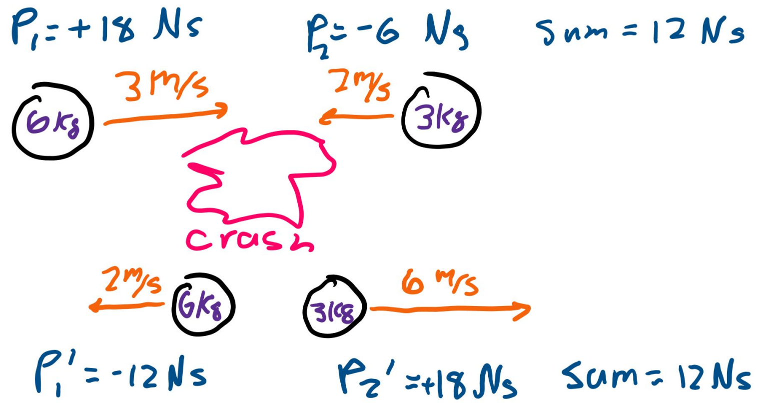
Since momentum is mv, and mass is usually constant, this rearranges to $F = mdv/dt = ma$.

However, by writing $F = dp/dt$, this allows us to account for a non-constant mass, which makes it a more general form of the law.

Conservation of momentum

Since change in momentum over time is force, if a system has no forces applied, the momentum should stay the same.

One instance of systems like this is a collision system.



Collisions

What are collisions

This is a way for multiple objects to interact while maintaining their momentums.

We will say initial momentums of each object are p_i for object i and the final momentum is p_i'.

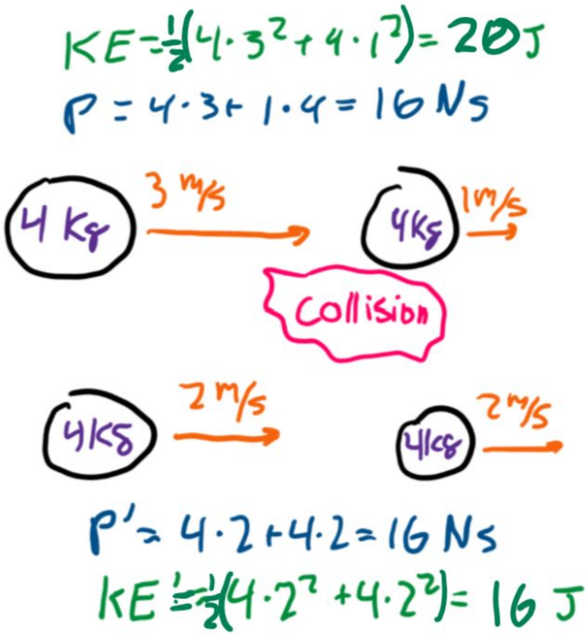
In collisions, multiple objects will collide with each other, and their velocities may change while the total momentum of the system is constant.

Inelastic

Most collisions are inelastic meaning objects will lose KE after colliding, but momentum will still remain the same.

The reason they lose KE is because friction acts on objects as they collide, turning KE to heat.

$p_1+p_2=p_1'+p_2'$.



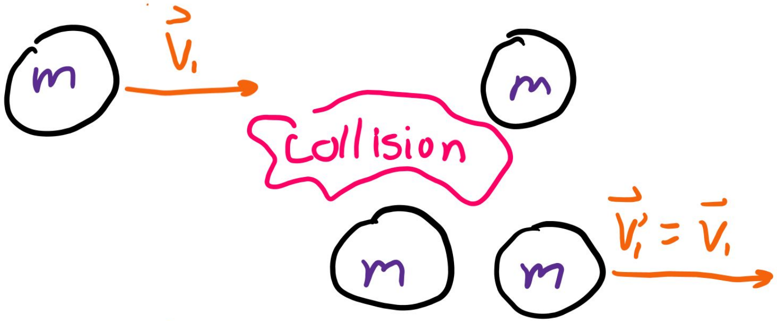
Elastic

An elastic collision means that not only is momentum conserved, but KE is as well.

These are elastic, as objects do not stick to each other, but bounce off each other after hitting.

$$KE = \frac{1}{2}mv_1^2 + \frac{1}{2}m0^2 = \frac{1}{2}mv_1^2$$

$$P = mv_1 + m\cdot 0 = mv_1$$



$$P' = m\cdot 0 + mV_1 = mV_1$$

$$KE' = \frac{1}{2}m0^2 + \frac{1}{2}mV_1^2 = \frac{1}{2}mV_1^2$$

Equations for elastic collisions

$p_1+p_2=p_1'+p_2'$ AND $KE_1+KE_2=KE_1'+KE_2'$.

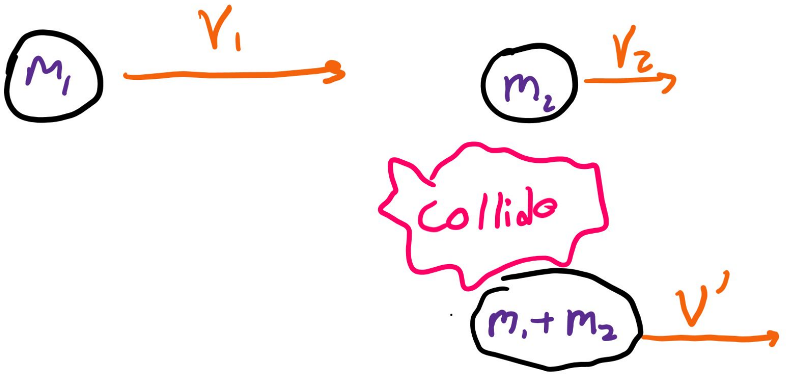
Rearranging gives $v_1+v_1'=v_2+v_2'$.

There's also a nice formula that gives each final velocity:
$v_1'=(m_1-m_2)v_1/(m_1+m_2) + (2m_2)v_2/(m_1+m_2)$

$v_2'=(2m_1)v_1/(m_1+m_2) + (m_2-m_1)v_2/(m_1+m_2)$

Perfectly inelastic

This is where colluding objects stick to each other, which means that the sum of momentums of the two objects p_1+p_2 turns into $p_{1,2}$, the momentum of the new object with combined mass of object 1 and 2.



Explosions (not a collision)

This is the “opposite” of a perfectly elastic collision where one object of momentum $p_{1,2}$ splits into two (or more) objects with momentum p_1 and p_2.